Nov 11th, 12:00 AM

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Eighth International Specialty Conference on Cold-Formed Steel Structures
St. Louis, Missouri, U.S.A., November 11-12, 1986

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BY

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ABSTRACT

In this paper an outline is given of a theoretical approach which is able to predict the equilibrium behaviour of thin-walled, pin-ended, I-section columns after local buckling. The extreme changes in cross-sectional shape that occur during overall bending of the locally buckled column are accounted for quite easily in the analysis by the inclusion of several local deflection shape functions. The changes in cross-sectional shape that occur during the pure compressional phase of behaviour, prior to overall bifurcation, are also accounted for in the analysis.

The Rayleigh-Ritz method is used to obtain local buckling loads and a semi-energy method is employed to describe post-buckling interaction behaviour. It is shown that, although all columns exhibit stable equilibrium behaviour after local buckling, the quality of the equilibrium at failure due to overall bifurcation, can be unstable to varying degrees depending on geometry. It is also shown that the theoretical predictions of equilibrium behaviour and ultimate loads are significantly influenced by the number of local deflection shape functions incorporated in the post-buckling analysis. If insufficient functions are employed in the solution, a considerable overestimate of stiffness and hence ultimate load can result.

The analysis is able to accommodate eccentricity of applied loading and column axis imperfection in which case, overall bending would occur from the onset of loading. A study is made of the effect of column axis imperfection on the ultimate carrying capability of thin-walled I-section columns and it is shown that, I-sections are appreciably sensitive to this mode of imperfection over a wide design range.

Results are presented in the paper in the form of load-deflection equilibrium curves and ultimate load-slenderness plots. A comparison of the theory with independent experimental work is also included and agreement is seen to be good.

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INTRODUCTION AND DISCUSSION OF SOME PREVIOUS WORK

In this paper the authors confine their attention to column flexural buckling, section post-local buckling and to the coupling or interaction between these two modes of behaviour. The theoretical analysis and discussion throughout the paper is presented with reference to a column whose local buckling load is less than its overall flexural buckling capacity. In view of this, it should be noted that there are certain basic differences in behaviour which exist between concentrically loaded columns with symmetric cross-sectional shape, such as I and box sections, and those with singly-symmetric cross sections such as plain and lipped channels. The fundamental difference is, that for columns with singly symmetric cross section, overall bending occurs from the onset of local buckling whereas for columns with symmetric cross section, overall deflections occur sometime after local buckling and are initiated by flexural bifurcation of the locally buckled column.

The reason for this difference in behaviour is due, of course, to the fact that for columns with singly symmetric cross section the change in cross sectional shape due to local buckling causes a shift in the neutral axis position of the section, in which case, an initially concentrically loaded column becomes effectively eccentrically loaded and hence both column and local deflections grow simultaneously with change in load from the onset of local buckling. In the case of symmetric cross sections however the change in cross sectional shape due to local buckling is such that no shift in the neutral axis of the section takes place and hence the column remains straight after local buckling and only the local deflections grow with increase in load until the beginning of column buckling.

Obviously, since the load-deflection equilibrium characteristics are quite dissimilar after local buckling for the two types of section mentioned, it is to be expected that their modes of failure will have distinctive characteristics also. A column with a singly symmetric cross section can fail at local buckling and the failure will be unstable and particularly sensitive to geometrical imperfections. This will occur, of course, if the design is in the range of near simultaneous local and overall flexural buckling. Outwith this range, however, the failure will be due to material yielding after some degree of stable post-local buckling and column bending interaction. In the case of columns with symmetric cross section the situation is quite different. Short strut lengths of column will fail due to material yielding or crippling after the section has undergone some amount of pure compressional post-local buckling behaviour. The failure of longer columns, on the other hand, in the intermediate slenderness range, is due to the tendency of the column to buckle in a flexural manner before crippling.

The equilibrium and failure characteristics discussed thus far, for the two types of section, have been investigated both theoretically and experimentally by numerous researchers in the past and such aspects as geometrical imperfections and material yielding have been considered and their effects highlighted. Typical theoretical reports concerned with singly symmetric cross sections are those by Rhodes and Harvey (12), Loughlan (9), and Loughlan and Upadhya (10), while those concerned with symmetric cross sections have been presented by Bijlaard and Fisher (2), Graves Smith (5), (6), and Hancock (7). Experimental investigations have been carried out into the post-locally buckled compressional behaviour of columns with symmetric cross section by Bijlaard and Fisher (1), Graves Smith (4), De Wolf et al (3) and Kalyanaraman et al (8). The experimental work of Thomasson (13), on the other hand, deals solely with columns having singly symmetric cross section and in particular with the effect on equilibrium behaviour of intermediate stiffeners formed integrally into the wide plate elements of the sections by the manufacturing process. In references (3), (8) and (13), theoretical considerations of the post-local buckling and column bending interaction problem have also been outlined.

In the theoretical works referred to, the methods of approach used by the individual researchers are indeed varied, as are the aspects considered by them to be of particular importance. Using their method of split rigidities, specific expressions were derived by Bijlaard and Fisher (2) for columns with I and square box sections, from which the ultimate strength in the post-local buckling range could be evaluated. In their paper, remarkably good agreement was shown between the ultimate stresses obtained from these expressions and those of numerous tests carried out in their experimental program. The effect of column axis imperfection was also considered in their work although it is clear from this analysis that their theory was quite unable to predict the unstable load-deflection equilibrium behaviour evident in so many of their tests, particularly for those columns with I cross section.

Hancock (7) studied the behaviour of I-section columns using a finite strip model incorporating local imperfections, Graves Smith (5), (6) on the other hand used a variational approach including the effects of local imperfections and plasticity to examine the behaviour of box columns, while De Wolf et al (3) and Kalyanaraman et al (8) employed effective section models based on the concept of effective width to determine the behaviour of rectangular box and I-section columns.

In all of these works, no account is given of post-bifurcation behaviour or to the effect of column bending due to eccentricity of applied loading or to column axis imperfection. This was considered by Thomasson (13) in his work on I section columns. The approach used by Thomasson was to set up the relevant differential equations of equilibrium for the locally buckled column in the bent state and to solve these in conjunction with satisfaction of the necessary compatibility requirements at the boundaries between locally buckled and unbuckled portions of the column. Due to his assumptions regarding local buckling however his analytical model is shown clearly to predict erroneous load-deflection equilibrium behaviour for some column geometries under concentric loading, and in fact his ultimate load-slenderness curves are contradictory to those of the well established and accepted curves of Bijlaard and Fisher (2) and Graves Smith (5) for this type of problem.

In this paper the behaviour of thin-walled I section columns is determined by solving the column differential equation of equilibrium for specified increments of compression applied at the nodes of the column central local buckle. For centroidally loaded geometrically perfect columns with singly symmetric cross section, the differential equation of lateral equilibrium is meaningful from the onset of local buckling. In the case of symmetric cross sections, however, the differential equation is relevant only from the onset of column bifurcation, sometime after local buckling, and the initial problem is to determine this bifurcation load. The approach used by Graves Smith (5) and Hancock (7) in their determination of buckling loads was to apply a small bending strain to the column at each increment of applied axial compression in order to ascertain the flexural stiffness of the locally buckled section. The approach used in this paper to determine the behaviour of concentrically loaded geometrically perfect columns is to consider that this may be closely approximated to by the solution of a column having a distinctly small eccentricity of applied loading. In this case the differential equation is appropriate from the onset of loading and due to the fact that the analysis presented herein is more than capable of prescribing the changes in local form that occur after local buckling, during both the compressional phase and more importantly the bending phase of behaviour caused by column bifurcation, it is shown that the method of approach adopted describes adequately the behaviour of the column and in particular is able to predict the unstable post-bifurcation behaviour inherent in some column designs.

GENERAL OUTLINE OF THEORETICAL APPROACH

After local buckling a centroidally loaded, geometrically perfect, pin-ended I-section column continues to carry increased loading in its straight configuration at a reduced axial stiffness. Failure then occurs due either to section crippling or, precluding torsional behaviour, to interaction flexural buckling. Any attempt to describe theoretically the sequence of events from local buckling to failure for either of these modes will of necessity be fairly complex. This is due, in the case of section crippling, to the effects of elasto-plastic deformations, and in the case of column interaction buckling to the considerable changes in cross sectional shape that take place at bifurcation. This paper deals theoretically with the mechanics of column interaction buckling whereby the behaviour of the column is determined from the solution of the differential equation of lateral equilibrium of the column in its bent and locally buckled state.

Figure 1 shows the column loaded eccentrically by an amount e along the symmetrical axis of the section parallel to its flanges. The column is in its bent and locally buckled state and it is assumed that a local buckle at the centre has a sinusoidal form along the member with half-wavelength λ. In this configuration the compression system at the nodes of the central buckle will be as shown in figure 2, in which case u is the compression of the section web and α is a compression eccentricity factor. Figure 3 shows, schematically, the average longitudinal stress system in the representative central portion of the column corresponding to this compression system and the areas B_1, B_2 and B_3 indicated, represent load losses due to local buckling in the flanges and web of the section. The method of analysis is to consider the total strain energy content of the central portion as distinct from the column as a whole, and to do this it is necessary to think of the centre portion as a short strut of length λ acted on externally by the force system shown in figure 3.

The local deflections w_i across the section are prescribed in terms of unknown coefficients and in accordance with the coordinate system shown in figure 4. To obtain the stress system within the section which complies with the displacements which take place during loading, use is made of von Karman's compatibility equation

$$\nabla^4(F_i) = \frac{\partial^4 F_i}{\partial x^4} + 2\frac{\partial^4 F_i}{\partial x^2 \partial y_i^2} + \frac{\partial^4 F_i}{\partial y_i^4} = E\left\{\left(\frac{\partial^2 w_i}{\partial x \partial y_i}\right)^2 - \frac{\partial^2 w_i}{\partial x^2}\cdot\frac{\partial^2 w_i}{\partial y_i^2}\right\} \qquad \ldots(1)$$

where F_i is a stress function for the ith plate such that

$$\sigma_x = \frac{\partial^2 F_i}{\partial y_i^2}, \quad \sigma y_i = \frac{\partial^2 F_i}{\partial x^2}, \quad \tau xy_i = -\frac{\partial^2 F_i}{\partial x \partial y_i} \qquad \ldots(2)$$

The local out of plane deflections w_i of each plate element are assumed to be of the form

$$w_i = Y_i(y_i) \cos \frac{\pi x}{\lambda} \qquad \ldots(3)$$

where x is the longitudinal distance from the column centre. The cross sectional deflected shape is taken as

$$Y_i(y_i) = \sum_{n=1}^{n=N} A_n Y_{in}(y_i) \qquad \ldots(4)$$

where, for each term n in the series, the function sets Y_{in} are compatible algebraic polynomials arranged to satisfy all the necessary boundary conditions at the plate junctions and at the free edges of the flanges.
To describe accurately the locally deflected shape at the point of buckling and to account adequately for the changes in local shape that occur, firstly, during the pure compressional phase of behaviour and then during the column bending phase, four terms are incorporated in the series of equation (4).

Substituting for w_i, using equations (3) and (4), into equation (1) and satisfying the relevant in-plane boundary conditions at the individual plate edges and the section ends, gives the middle surface stresses for the section in terms of u, α and the magnitude coefficients A_n of the locally deflected shapes. The final equations relating end movements, membrane stresses and out of plane displacements are obtained using the principle of minimum strain energy. The total elastic strain energy stored in the section during local buckling is obtained in terms of the stress functions F_i and the deflection functions w_i from the following expression

$$V_T = \sum \iint \frac{D_i}{2} \left\{ \left[\frac{\partial^2 w_i}{\partial x^2} + \frac{\partial^2 w_i}{\partial y_i^2} \right]^2 - 2(1-\nu) \left[\frac{\partial^2 w_i}{\partial x^2} \cdot \frac{\partial^2 w_i}{\partial y_i^2} - \left(\frac{\partial^2 w_i}{\partial x \partial y_i} \right)^2 \right] \right\} dxdy_i$$
$$+ \sum \iint \frac{t_i}{2E} \left\{ \left[\frac{\partial^2 F_i}{\partial x^2} + \frac{\partial^2 F_i}{\partial yi^2} \right]^2 - 2(1+\nu) \left[\frac{\partial^2 F_i}{\partial x^2} \cdot \frac{\partial^2 F_i}{\partial y_i^2} - \left(\frac{\partial^2 F_i}{\partial x \partial y_i} \right)^2 \right] \right\} dxdy_i \qquad \ldots(5)$$

Substituting for w_i as described by equations (3) and (4) and F_i from the solutions of equation (1) into equation (5) the total strain energy is then minimised with respect to the deflection magnitude coefficients A_n to produce the required relationships. From the analysis, and with reference to figures 2 and 3, equations for the axial load on the section and the moment about the section web are obtained as follows

$$P = 2\left(\frac{2EU}{\lambda}\right)\left[\left(1 - \frac{\alpha}{2}\right)b_1t_1 + \left(1 + \frac{\alpha}{2}\right)b_2t_2 + b_3t_3\right] - 2\left(B_1 + B_2 + B_3\right) \qquad \ldots(6)$$

$$M = \left(\frac{2EU}{\lambda}\right)\left[\left(1 - \frac{2}{3}\alpha\right)b_1{}^2t_1 - \left(1 + \frac{2}{3}\alpha\right)b_2{}^2t_2\right] - \left(\bar{B}_1 - \bar{B}_2\right) \qquad \ldots(7)$$

where the load losses B_i are described by

$$B_i = \frac{E\pi^2 t_i}{4\lambda^2}\int_0^{b_i} Y_i{}^2 dy_i \qquad \ldots(8)$$

and the moments $\bar{B}_i$, about the section web, due to these losses by

$$\bar{B}_i = \frac{E\pi^2 t_i}{4\lambda^2}\int_0^{b_i} Y_i{}^2 y_i dy_i \qquad \ldots(9)$$

For the case of an I-section column with overall imperfection δ_o, the governing differential equation for the column in its bent and locally buckled state is

$$EI^* \frac{d^2(\delta - \delta_o)}{dx^2} + P\delta = -P\left[e - d^* + \frac{M^*}{P}\right] \qquad \ldots(10)$$

The parameter d^* in this equation defines the effective position of the section neutral axis in this buckled configuration, I^* is a reduced second moment of area of the section which takes local buckling into account and M^* is an internal moment caused by the effects of local buckling. Expressions for these parameters are given in reference 11.

Assuming that column axis imperfections may be described by

$$\delta_o = \delta_{oc} \cos \frac{\pi x}{\ell} \qquad \ldots(11)$$

then the solution of equation 10 is

$$\delta = \left[e - d^* + \frac{M^*}{P}\right]\left[\sec \frac{k\ell}{2} \cos kx - 1\right] + \frac{\delta_{oc}}{1 - \frac{P}{P_E*}} \cos \frac{\pi x}{\ell} \qquad \ldots(12)$$

where $k^2 = P/EI^*$ and P_E* is a reduced Euler load given by $P_E* = \pi^2 EI^*/\ell^2$. The central deflection of the column can be obtained by setting $x = o$ in equation 12 and can be expressed in the form

$$\delta_c = \left[e - d* + \frac{M^*}{P}\right]\left[\sec \frac{\pi}{2}\sqrt{\frac{P}{P_E*}} - 1\right] + \frac{\delta_{oc}}{1 - \frac{P}{P_E*}} \qquad \ldots(13)$$

The external moment about the section web at the centre of the column is obtained, with reference to figure 1, as

$$M = P(e + \delta_c) \qquad \ldots(14)$$

Equating this to the internal moment given by equation 7 and then substituting for P and δ_c from equations (6) and (13) gives the single equilibrium equation for the column as a function of the eccentricity of compression factor α in the form

$$f(\alpha) = 0 \qquad \ldots(15)$$

This equation is solved numerically to obtain the equilibrium value of α for any given web compression u using the solution procedure outlined in reference 9. The solution is now complete for this particular value of u and the complete equilibrium path for the column is determined by repeating the process for a number of discrete increments of compression u applied to the section web.

The method of approach adopted in this paper has been applied, with considerable success, to determine the column interactive buckling behaviour of pin-ended plain and lipped channel sections in references 9, 10 and 11. For these sections the differential equation 10 is appropriate from the onset of local buckling since the shift in section neutral axis, caused by local buckling, means effectively that the section becomes eccentrically loaded thus introducing column bending at the instant of local buckling.

The changes in cross section shape due to column bending after local buckling are small for lipped channel sections and these have been adequately accounted for in reference 9 in the direct manner used in this paper with only two terms utilised in the post-local buckling interactive analysis. The changes in cross section shape after local buckling can be considerable, depending on geometry, for plain channel sections and these have been accounted for in references 10 and 12 in an indirect manner by superposing a series of one term post-local buckling interactive solutions corresponding to different locally buckled shapes and choosing the lowest envelope of all such curves as being the closest approximation to the true solution. In the case of pin-ended centroidally loaded I-section columns, changes in the locally deflected shape are moderate during the pure compressional phase of behaviour but at the point of interaction buckling, the locally deflected shape changes quite markedly. This is due to increased local buckling occurring on the compression side of the column during overall bending and to local unbuckling on the tension side. To account adequately for these changes it was found necessary, depending on geometry, to utilise three or four terms in the post-local buckling interaction analysis in order to provide sufficient flexibility in the solution to obtain a good approximation of the behaviour of I-section columns.

As might be expected the effect of column bending will be to introduce a variation in the magnitude of the local buckles along the column. The local buckle magnitudes will be more pronounced at the centre of the column and will diminish along the member away from the centre. As pointed out in reference 10 the method of approach used in this paper leads to a form of local deflection amplitude modulation and on examination of the relevant equations it has been found that the local deflections do in fact diminish along the column.

RESULTS AND DISCUSSION

Figure 5 shows the load-deflection equilibrium behaviour of a range of I-section columns with flange-web width ratio $b_1/b_w = 0.4$ and flange-web thickness ratio $t_1/t_w = 0.5$. From a consideration of different lengths of this section curves are presented in the non-dimensional form of P/P_E against $\delta c/tw$ for a fairly wide range of the ratio of local buckling to Euler buckling load P_c/P_E. The curves are obtained from purely elastic considerations and in order to facilitate the use of the solution of the column differential equation during the interactive phase of behaviour of concentrically loaded columns it is necessary to model these with an extremely small value of load eccentricity. It can be seen from figure 5 that all of the columns carry increased load after local buckling and that column deflections are small during the compressional phase of behaviour but increase rapidly at column buckling. It is also of note that the nature of the equilibrium behaviour at column buckling is unstable for most of the columns with this cross sectional geometry and that the severity of the unloading process increases as the ratio of P_c/P_E tends to unity. Due to this it can be expected that geometrical imperfections will have a significant effect on failure and it is clear that the inclusion of imperfection modes in analysis would provide results more representative of the behaviour of actual columns. In this paper a study is made of the effect of column axis imperfection on failure and the reduction in load carrying capability is shown to be significant for this mode.

Figure 6 shows the load-deflection equilibrium behaviour of columns with the cross sectional parameters of $b_1/b_w = 0.5$ and $t_1/t_w = 0.5$. Two lengths of this cross section were considered such that $P_c/P_E = 0.75$ and 0.25. The full lined curves are from the theoretical analysis presented herein and the broken curves are those from the approximate analysis proposed by Thomasson (13). It is clear that for the case of $P_c/P_E = 0.75$ the equilibrium at column buckling is unstable and the effect of introducing small eccentricities of the applied loading is to reduce considerably the ultimate carrying capacity of the column. Two values of the eccentricity parameter $\bar{e} = e/b_1$ of 0.001 and 0.005 have been considered for this column and it is of note that the lower value results in an equilibrium curve similar in form to that of the concentrically loaded column but with a reduced failure load while the larger value results in failure and immediate unloading at local buckling. In contrast to these curves the approximate solution proposed by Thomasson suggests failure and immediate unloading from the onset of local buckling for the concentrically loaded column. For the case of $P_c/P_E = 0.25$ it is seen that, for centroidal loading, column buckling takes place at about just over twice the local buckling load and that the state of the equilibrium is practically neutral for this design. In contrast to this behaviour Thomasson suggests that column deflections grow from the onset of local buckling and that the equilibrium behaviour is stable. This is typical of the behaviour prescribed by columns with singly symmetric cross section but as has been pointed out and as has been shown for this design it is not typical of the behaviour of columns with symmetric cross section. The effect of load eccentricity on the equilibrium behaviour of this column design is evident from figure 6. Three values of the eccentricity parameter $\bar{e}$ of 0.01, 0.025 and 0.1 have been considered. Due to eccentricity and to the subsequent loss in section flexural stiffness caused by local buckling, considerable column deflections are seen to be experienced at loads well below the column buckling load and due to this it is to be expected that premature collapse of the column will occur due to material failure caused principally by column bending. Since load eccentricity and column axis imperfection have essentially the same effect on equilibrium and failure this fact will be corroborated later in comparisons between theoretical ultimate loads and those obtained from actual tests.

For a pin ended centroidally loaded I-section column the transition from the purely compressional phase of behaviour to that of column bending at overall bifurcation is accompanied by considerable changes in the locally deflected form of the cross section. It is clear then that any analytical approach to this problem has to be capable of prescribing these changes in order that meaningful predictions of the column's equilibrium behaviour and ultimate carrying capacity can be made. The changes in local form that occur during both the compressional and bending phases of behaviour are accounted for in the analysis presented herein in a fairly straightforward and direct manner through the provision of a sufficient flexibility of choice in the energy minimisation process of a number of local deflection shape functions for the section, each of which, satisfies independently, all the necessary equilibrium and compatibility requirements at the junctions between the connected plates of the section and at the cross section free edges.

Figure 7 shows the load-deflection equilibrium behaviour of the column discussed in figure 6 with $P_c/P_E = 0.75$ for four values of the load eccentricity parameter $\bar{e}$ of 0.0001, 0.0005, 0.001 and 0.0025. The broken curves are those associated with two shape functions in the analysis after local buckling and the full curves represent the behaviour predicted using three shape functions. It is clear that the difference between the two solutions is significant and that the two term solution predicts a much stiffer section with correspondingly higher ultimate loads than those predicted by the three term solution. Of particular interest is the loading case of $\bar{e} = 0.0025$ where it can be seen that the two term solution suggests a large loss of section flexural stiffness at local buckling but that the equilibrium behaviour remains stable and the column is able to sustain a small load increase before failure. In contrast to this it is seen that the three term solution indicates failure at local buckling and that the equilibrium behaviour is unstable with a high degree of severity in the unloading rate. It is evident that the use of two terms in the solution is insufficient in this case for the analysis to represent the column's true behaviour and the significance of incorporating a sufficient number of suitable local deflection shape functions in the analysis is obvious. The difference between the behaviour obtained from a four term solution and that of the three term solution was found to be very small and for this reason it may be said that the full curves of figure 7 are fairly accurate representations of the true elastic behaviour of the column.

Since load eccentricity and column axis imperfection have essentially the same effect on column behaviour a form of imperfection sensitivity curve based solely on load eccentricity has been constructed in figure 8 for the column discussed in the previous two figures. The base curve signifies local buckling and the upper full line curve is the ultimate load prediction based on a three term solution while the broken curve illustrates the ultimate load prediction based on a two term solution. It is of note that the two term solution suggests failure at local buckling for values of $\bar{e} > 0.0032$ whereas the three term solution indicates that failure occurs in this way for values of $\bar{e} > 0.0025$. The difference between the two solutions is evident and is seen to be greater for smaller values of $\bar{e}$. The reason for this is due, of course, to the fact that the local buckling mode for the smaller values of $\bar{e}$ is more closely related to that of the mode for pure compression and that during the transition from the pure compressional mode to that of column bending the considerable changes in shape that occur are more readily accounted for by the three term solution. From figure 8 it can be seen that the column in question is particularly imperfection sensitive. A value of $\bar{e}$ = 0.005 for example, is seen to result in failure at local buckling and a reduction of the ultimate load of the concentrically loaded column of the order of 18%. This is due of course to the unstable nature of the equilibrium of this column at bifurcation as discussed earlier and depicted in figure 6.

Precluding torsional instability, the failure of geometrically perfect thin walled I section columns will occur by cross section crippling caused by material yielding at the section junctions after local buckling for the smaller slenderness ratios, by flexural buckling for the higher slenderness ratios or by flexural buckling after local buckling for the intermediate slenderness ratios. The ultimate load levels associated with these three distinct modes of failure will be considerably reduced in real columns due to the effect of geometrical imperfections and the mode of failure in real columns, due particularly to column axis imperfections, generally divides into two distinct areas, that of local buckling for slenderness values above a certain critical value, depending on geometry and imperfection magnitude, and that of material yielding in the column bending mode after local buckling for slenderness values lower than the critical. Since the analysis presented herein is purely elastic, failure in the crippling mode for perfect columns is considered to occur when the maximum membrane stress at the section junctions reaches the material yield stress and failure in the column bending mode after local buckliing for imperfect columns is considered to occur when the maximum membrane stress at any point in the section reaches the material yield stress. These two yield criteria are shown in this paper to provide very good comparisons with test ultimate load values without having to resort to the complexities of a forbidding elasto-plastic analysis.

Figure 9 illustrates the failure characteristics for a column with b_1/b_w = 0.534 and t_1/t_w = 0.5. The average stress at failure is plotted against column slenderness and curves are shown for the geometrically perfect column and for a column with an axis imperfection amplitude of δ_{oc}/t_w = 1.0. The cross sectional parameters of this column are in fact those of the test columns LC III of the experimental program carried out at Cornell University and reported in the paper by Kalyanaraman et al (8) and comparisons with the present theory and these tests are shown and discussed later. The three distinct modes of failure of crippling, interaction buckling and Euler buckling for the perfect column are indicated and it is clear that the ultimate loads of the idealised column are unlikely to be achieved in practice. The effect on local buckling of column axis imperfection is evident and is seen to be considerable for the column design with simultaneous local and Euler buckling modes. It is also of note that failure is initiated by local buckling for this column cross section and column axis imperfection magnitude when L/r is greater than about 116 and for slenderness values less than this, failure is due to material yielding in the column bending mode after local buckling. The ultimate load sensitivity of this cross section to column axis imperfection is seen to be considerable over a fairly wide design range and is not just associated with those designs in the region of near simultaneous local and Euler buckling. The greatest sensitivity is found to occur over the slenderness range spanned by the two designs of simultaneous local and Euler buckling and simultaneous crippling and Interaction buckling. For the imperfection magnitude of δ_{oc}/t_w = 1.0 considered, the reduction in carrying capability for these two designs is shown to be 34.6% and 34.4% respectively. For a design situated between these two and with a slenderness value of $L/r = 108$ the analysis suggests a reduction in carrying capacity of the order of 38.5%. For comparative reasons the section squash load $P_y = A\sigma_y$ is also depicted in figure 9 and it is seen that due to local buckling, the actual failure load of the section in the crippling mode is of the order of 18% lower than the squash load.

COMPARISON WITH EXPERIMENT

Tests performed at Cornell University on steel I-section columns, manufactured by connecting cold formed plain channels back to back, have been reported by Kalyanaraman et al (8). The ultimate loads of the short stub columns from the SC series of tests and of the longer columns from the LC series of tests are compared with those predicted from the theory presented in this paper. Theoretical comparisons with ultimate loads from the LC series of tests have been made by Kalyanaraman et al (8) who used an effective section approach based on empirically derived effective width expressions for the individual component plates of the section and by Hancock (7) who used a finite strip model with local plate imperfections. Both models precluded the effect of load eccentricity or column axis imperfection and produced interaction buckling loads which were in fairly close agreement with the ultimate loads obtained from the LC series of tests. Measurements of local and column axis imperfections were not reported for the LC test series although it was considered that the columns had a minimal out of straightness and were loaded concentrically. The local imperfection magnitude considered in the analysis by Hancock (7) was assumed on the basis of measurements taken in an earlier test series by De Wolf et al (3) and the results of these tests are also reported in reference (8).

The analysis presented in this paper precludes the effect on column behaviour of local imperfections and concentrates solely on that of load eccentricity or column axis imperfection. It is suggested also at this point that perhaps a more realistic local imperfection mode for I-section columns, as an alternative to the symmetric mode used directly in the analysis by Hancock (7) and indirectly in the analytical model postulated by Kalyanaraman et al (8), would be one of a non-symmetric nature which introduced bending of the column from the onset of loading.

The effect of column axis imperfection on the load carrying capacity of the columns from the LC test series are shown in figures 10, 11, 12, 13 and 14 and it is clear from these that the test columns are indeed sensitive to this mode of imperfection. The test columns were designed to give a post-buckled failure ratio, based on section crippling, of between 2.76 for the SCI and LCI tests depicted in figure 10 to about 1.04 for the SCV and LCV tests shown in figure 14. Arbitrary values of the overall imperfection parameter δ_{OC} have been chosen to give comparison with the ultimate loads from test and it is of note that, with the exception of the LCI tests depicted in figure 10, an imperfection magnitude of $\delta_{OC}/t_w = 1.0$, or less, is sufficient to provide favourable agreement. The theoretical average stresses at failure due to section crippling are seen to compare fairly well with the ultimate stress values obtained from the stub-column tests, although an overestimate on failure is predicted for the SCV specimens shown in figure 14 for which the ratio of crippling stress to local buckling stress is close to unity. According to the present theory, the test columns LCI and LCII were designed to fail either by section crippling or by column interaction buckling as shown in figures 10 and 11 respectively. The slenderness range of each test set is seen to be removed from the near simultaneous local and Euler buckling range and to straddle the slenderness value for which simultaneous crippling and column interaction buckling would occur. The sensitivity to column axis imperfection is seen to be significant over the whole of the interaction buckling range and in fact is noted to be greater for a simultaneous crippling and interaction buckling design than for a simultaneous local and Euler buckling design. In the near simultaneous local and Euler buckling design range, failure is seen to be initiated by local buckling for the geometrically imperfect columns.

It is clear from figures 10 and 11 that the test columns LCI and LCII did in fact have geometrical imperfections and it is highly probable that the nature of these was some combination of both the local and overall imperfection modes. It is also highly likely that any local imperfections present were not symmetrically disposed across the section, thus serving to promote column bending from the onset of loading and failure due to material yielding in the column bending mode. In the absence of information relating to imperfection measurements for the LC test series, the ultimate stress curves shown in figures 10 and 11 for the geometrically imperfect columns are intended, mainly, to indicate the sensitivity of the test columns to the column axis imperfection mode and also to give some form of comparison with the ultimate stress levels obtained from test. An imperfection magnitude of $\delta_{oc}/t_w = 1.0$ is seen to give very good agreement with the LCII test columns shown in figure 11.

The slenderness values of the three columns from the LCIII test set are such that, according to the theory for geometrically perfect columns, they would provide failure modes of section crippling, column interaction buckling, and almost simultaneous local and Euler buckling as indicated in figure 12. The actual failures however are seen to correspond to considerably lower ultimate average stresses than those predicted from the geometrically perfect theory and an overall imperfection magnitude of $\delta_{oc}/t_w = 1.0$ is again seen to give very good agreement with the tests. It is fairly evident from figure 12 that the LCIII test columns were geometrically imperfect and again it is probable that some combination of both local and overall imperfections were present. The failure curve shown in figure 12 based on the arbitrarily chosen value of the imperfection parameter $\delta_{oc}/t_w = 1.0$ indicates the considerable sensitivity of the LCIII columns to the column axis imperfection mode and, coincidentally, provides very favourable agreement with the ultimate average stresses of the test columns.

Figures 13 and 14 show the comparison between theory and experiment for the LCIV and LCV columns respectively. The post-local buckled failure ratio, based on section crippling, is noted to be lower for these columns than those considered earlier. It is of the order of 1.25 for the LCIV columns and about 1.04 for the LCV specimens. It is of note that all of the LCIV and LCV test columns designed to buckle locally, fail at average stress values lower than the local buckling stress of the geometrically perfect column and that imperfection magnitudes of δ_{oc}/t_w less than unity are sufficient to give reasonable failure predictions. As expected the lower post-local buckled failure ratios based on section crippling are seen to result in a higher degree of imperfection sensitivity for these columns. The ultimate failures shown in figure 13 for the geometrically imperfect columns are noted to be due to local buckling for slenderness values greater than about 90 for the imperfection magnitude of $\delta_{oc}/t_w = 0.7$ and for the slenderness values greater than about 95 for the imperfection magnitude of $\delta_{oc}/t_w = 1.0$. Due to the closeness of the local buckling stress and the material yield stress of the LCV test columns, the failure curves shown in figure 14 for geometrically imperfect columns are the average column stresses corresponding to section first yeild in the column bending mode prior to the occurrence of local buckling. The curves have been obtained using simple engineering theory and it is seen that imperfection magnitudes of δ_{oc}/t_w of between 0.3 and 0.6 would provide fairly good agreement with test. Since the column interaction buckling range is almost completely eliminated when local buckling and material yielding are almost coincident, the highest degree of imperfection sensitivity is noted, in this case, to be concentrated in the slenderness range of near simultaneous local and Euler buckling.

CONCLUSIONS

It has been shown that the theoretical approach outlined in this paper is able to predict the interaction buckling loads and post interaction buckling behaviour of thin-walled pin-ended I-section columns and that for the geometries considered the theory has indicated that the nature of the equilibrium at buckling is unstable and therefore sensitive to the effects of geometrical imperfections.

In the analysis, full interaction between the individual plate elements of the cross section is accounted for and the effect of allowing the locally deflected form to change with load is also included. It has been shown that if insufficient local deflection shape functions are incorporated in the analysis, the imposed restriction on solution flexibility significantly influences the theoretical predictions of equilibrium behaviour and ultimate load.

The analysis considers the effect of load eccentricity or column axis imperfection and it has been shown that the effect of these is to undermine, considerably, the theoretically determined interaction buckling loads of geometrically perfect columns. Unlike the behaviour of pin-ended columns with singly symmetric cross sections, for which it has been shown, in reference (9) for example, that the most severe sensitivity to column axis imperfection occurs for designs with simultaneous local and Euler buckling, the behaviour of I-section columns is such that the degree of imperfection sensitivity can be significant over a much wider design range.

If the cross sectional dimensions of the I-section provide a high post-local buckled failure ratio based on section crippling, this results in a wider slenderness range over which column interaction buckling occurs and hence over which column axis imperfections will be significantly effective. As the post-local buckled failure ratio based on crippling reduces, however, the greatest sensitivity to column axis imperfections then becomes more concentrated in the near simultaneous local and Euler buckling design range.

The simple collapse criterion of assuming failure to occur when the maximum membrane stress in the section, over the central portion of the column, reaches the yield stress of the material has been shown to give reasonable agreement with the failure loads of independent column tests. It is to be recognised that the column axis imperfection magnitudes considered in the paper are purely arbitrary values which serve only to indicate, theoretically, the sensitivity of I-section columns to this mode of imperfection and to give some form of comparison, in the absence of relevant imperfection data, with the independent column tests carried out at Cornell University.

The theory precludes the effect on column behaviour of local imperfections but it is able to consider the simultaneous development of column bending and local buckling deflections with load, thus permitting a study of the post interaction buckling equilibrium behaviour of I-section columns.

Appendix.--References

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Appendix.--Notation

B_1, B_2, B_3	=	Load losses in Individual Plate Elements.
$\bar{B}_1, \bar{B}_2$	=	Moments due to Load Losses.
$b_i = b_1, b_2, b_3$	=	Widths of individual Plate Elements.
D_i	=	Flexural Rigidity of plate i.
d^*	=	Effective Position of Neutral Axis Relative to Web.
E	=	Young's Modulus of Elasticity.
e	=	Eccentricity of Applied Load.
F_i	=	Stress Function for Plate i.
I^*	=	Reduced Second Moment of Area.
L	=	Column Length.
M	=	Moment about web.
M^*	=	Internal Moment caused by Local Buckling.
P	=	Applied Load.
P_E^*	=	Reduced Euler Load.
$t_i = t_1, t_2, t_3$	=	Plate thicknesses. ($t_3 = t_w = 2t_1$)
u	=	Compression of web at Nodes of Central Buckle.
V_T	=	Total Strain Energy.
w_i	=	Local Deflections.
x	=	Coordinate along Column from Column Centre.
y_i	=	Coordinate across Plate Element i
α	=	Eccentricity of Compression Parameter.
δ	=	Overall Column Deflections.
δ_o	=	Overall Imperfections.
δ_c	=	Overall deflection at Column Centre.
δ_{oc}	=	Overall Imperfection at Column Centre.
λ	=	Local Buckle half-wavelength.
ν	=	Poisson's Ratio.
σ_x	=	Longitudinal Stresses.
σ_{yi}	=	Stresses Normal to the Longitudinal Direction.
τ_{xy_i}	=	Shear Stresses.

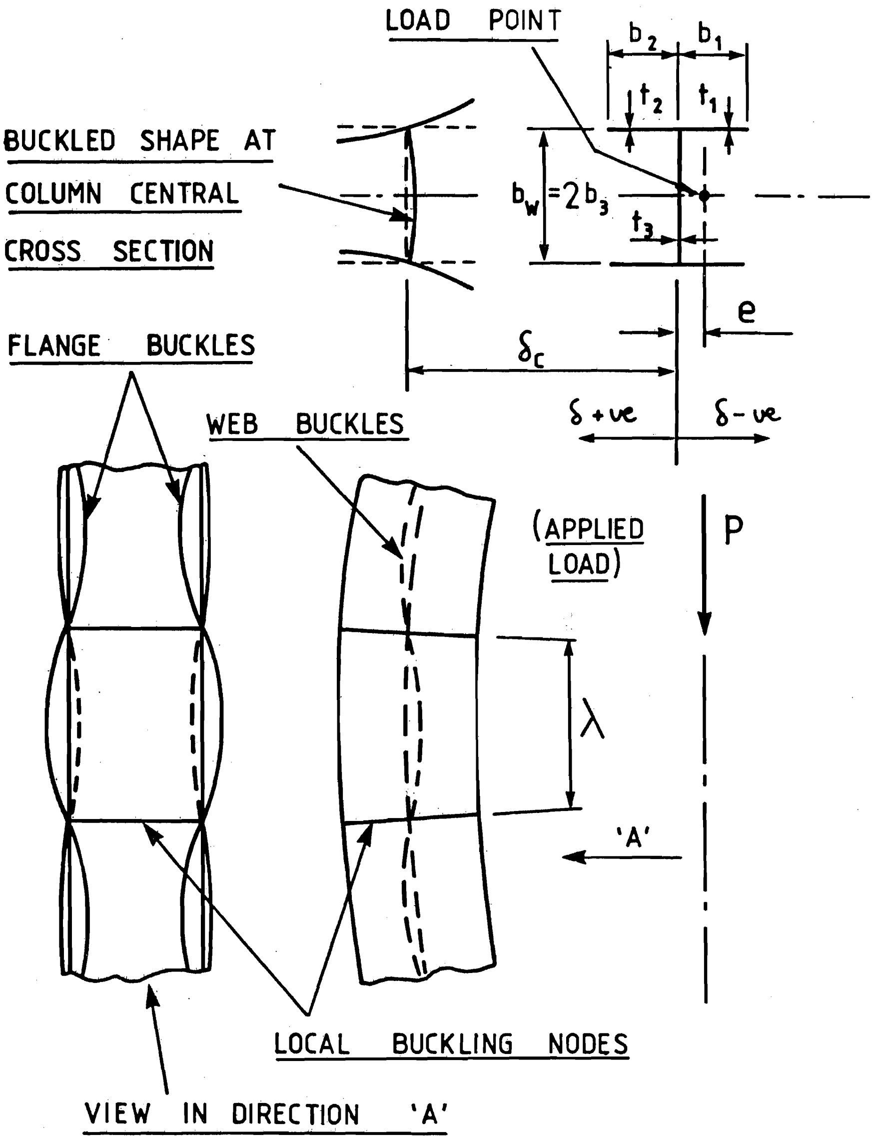

FIGURE 1. INTERACTIVE BUCKLING MODE OVER CENTRAL PORTION OF COLUMN.

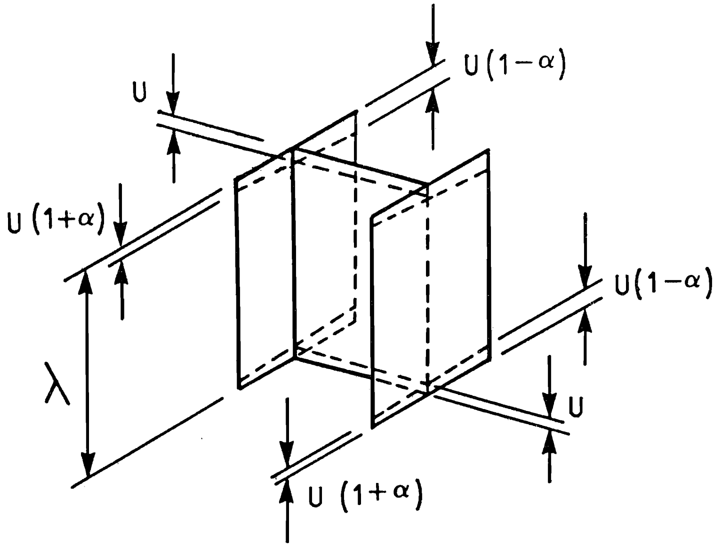

FIGURE 2. COMPRESSIVE DISPLACEMENTS AT NODES OF CENTRAL BUCKLE

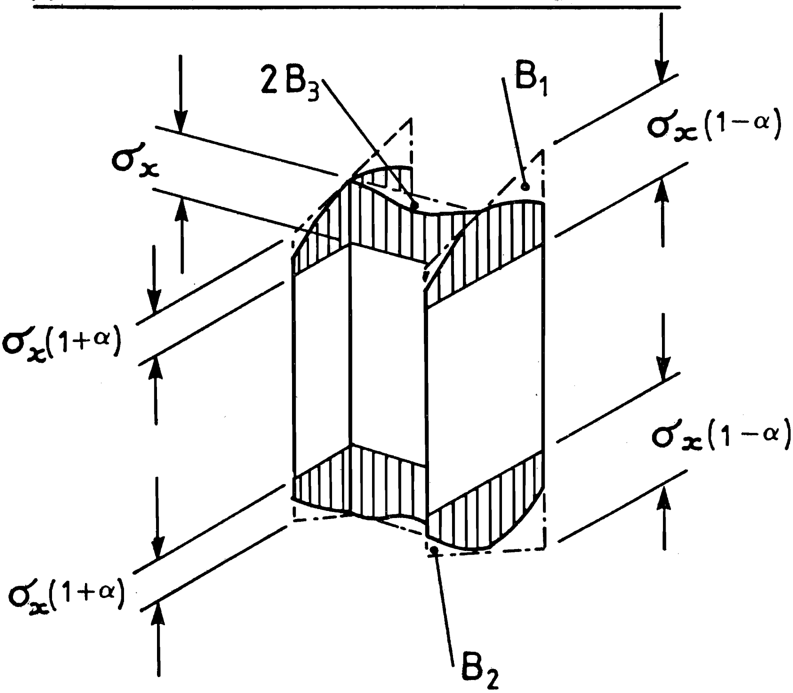

FIGURE 3. SCHEMATIC REPRESENTATION OF AVERAGE LONGITUDINAL STRESSES AFTER LOCAL BUCKLING.

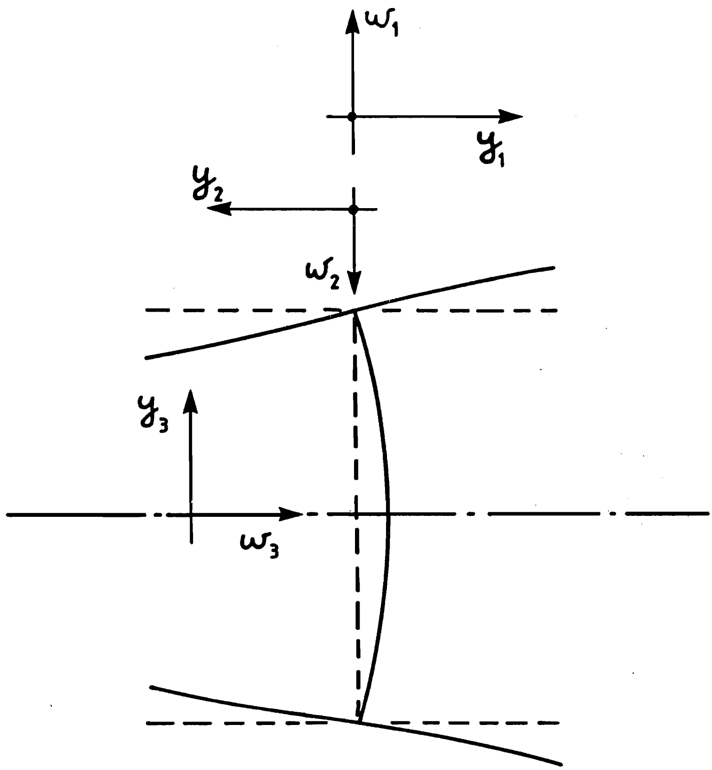

FIGURE 4. COORDINATE SYSTEM FOR LOCAL DEFLECTIONS.

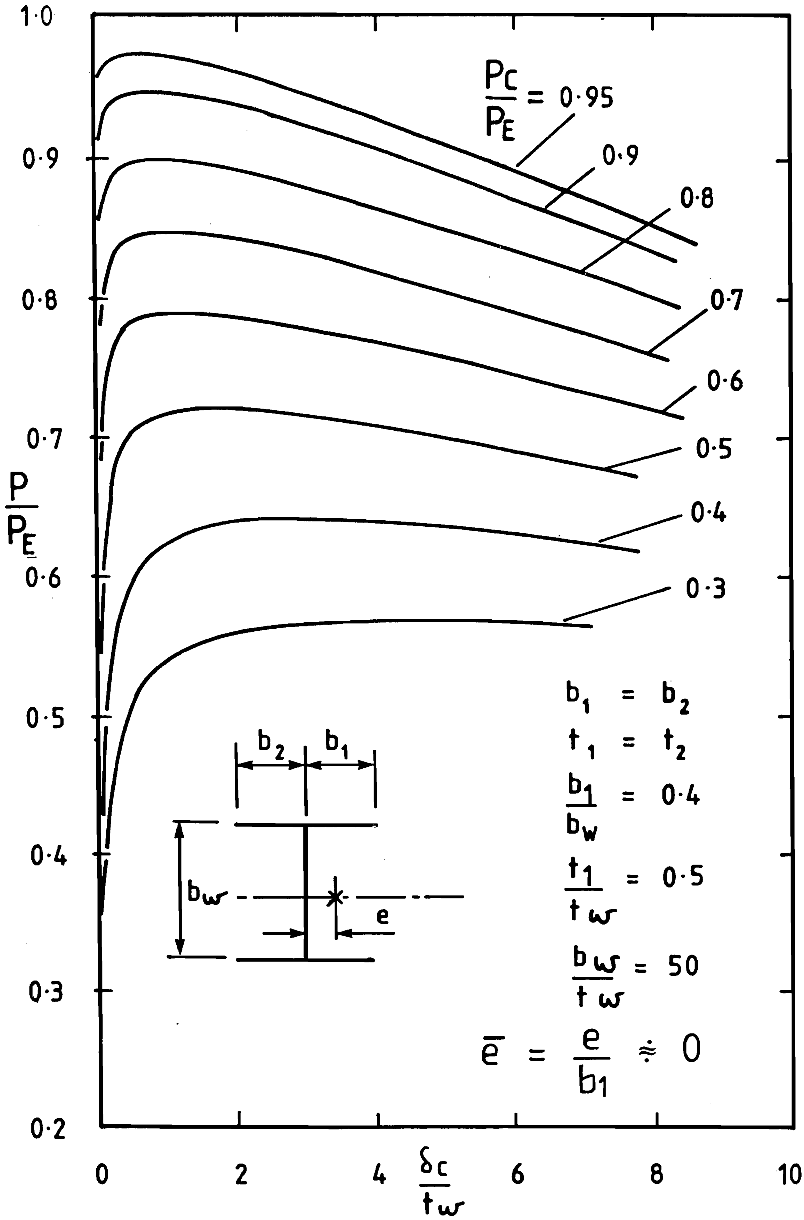

FIGURE 5. EFFECT OF COLUMN LENGTH ON INTERACTION BUCKLING EQUILIBRIUM BEHAVIOUR.

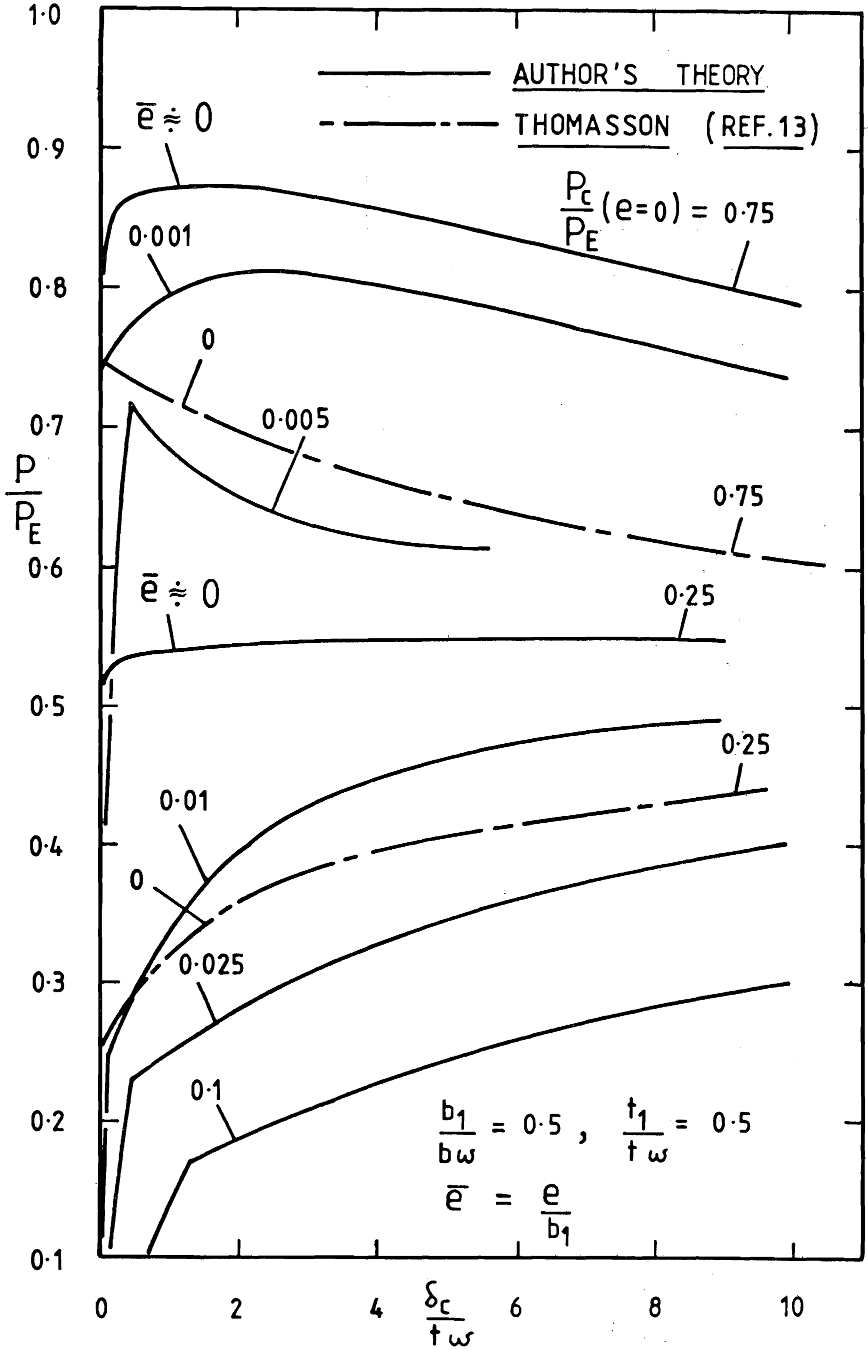

FIGURE 6. EFFECT OF LOAD ECCENTRICITY.

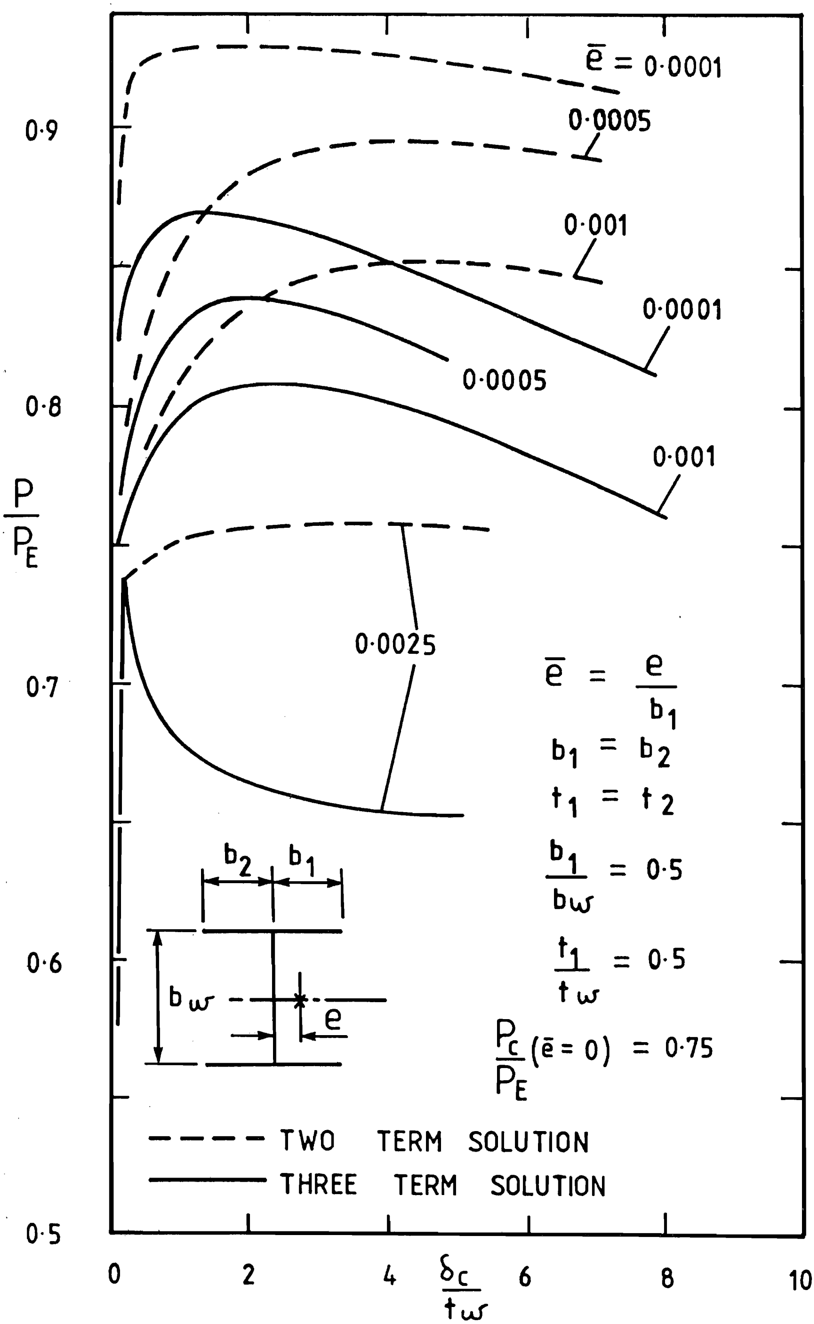

FIGURE 7. EFFECT OF SOLUTION FLEXIBILITY.

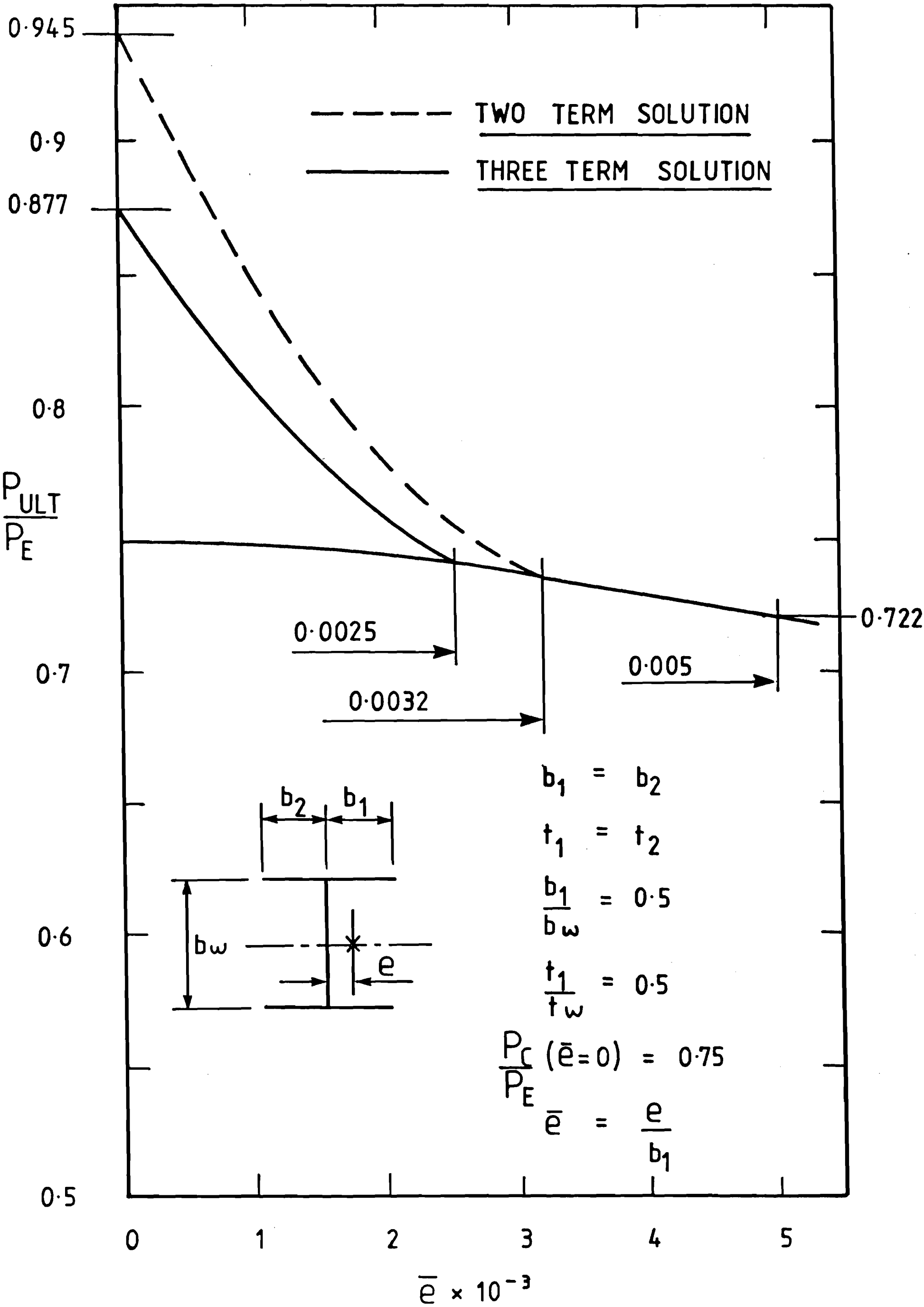

FIGURE 8. IMPERFECTION SENSITIVITY.

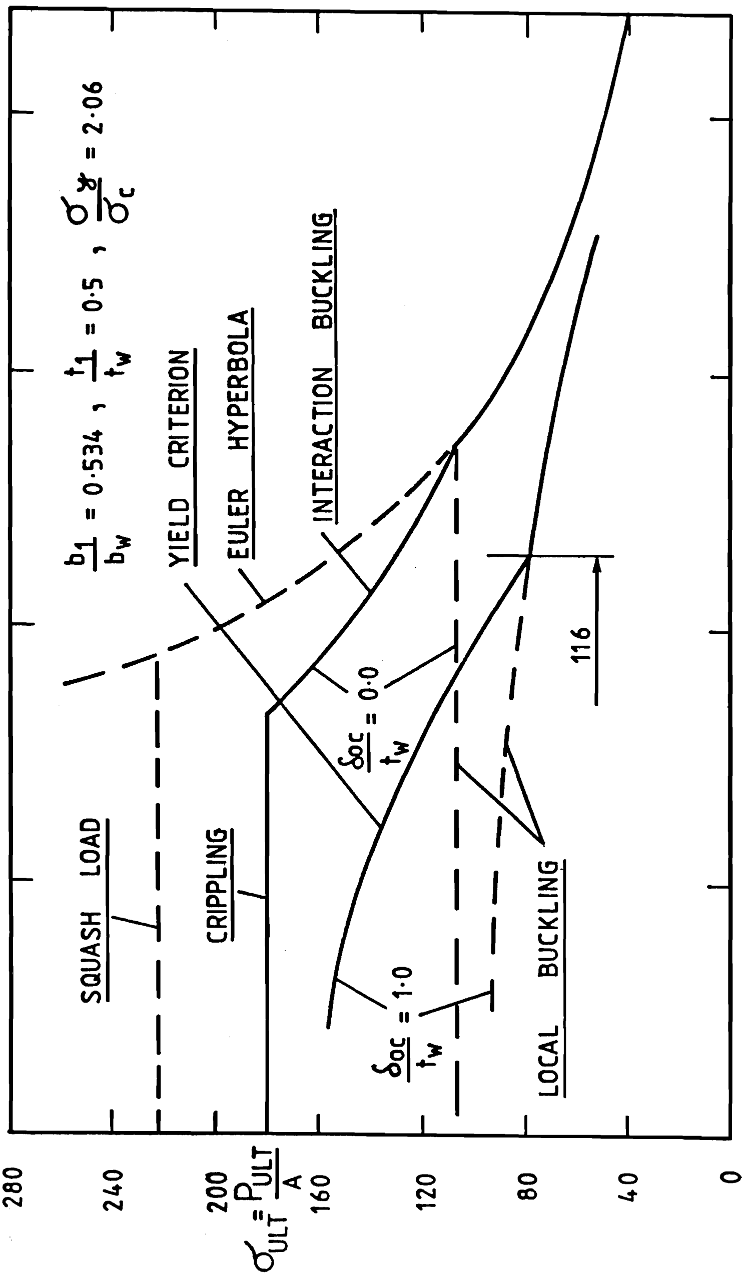

FIGURE 9. ULTIMATE AVERAGE FAILURE STRESS VERSUS SLENDERNESS RATIO.

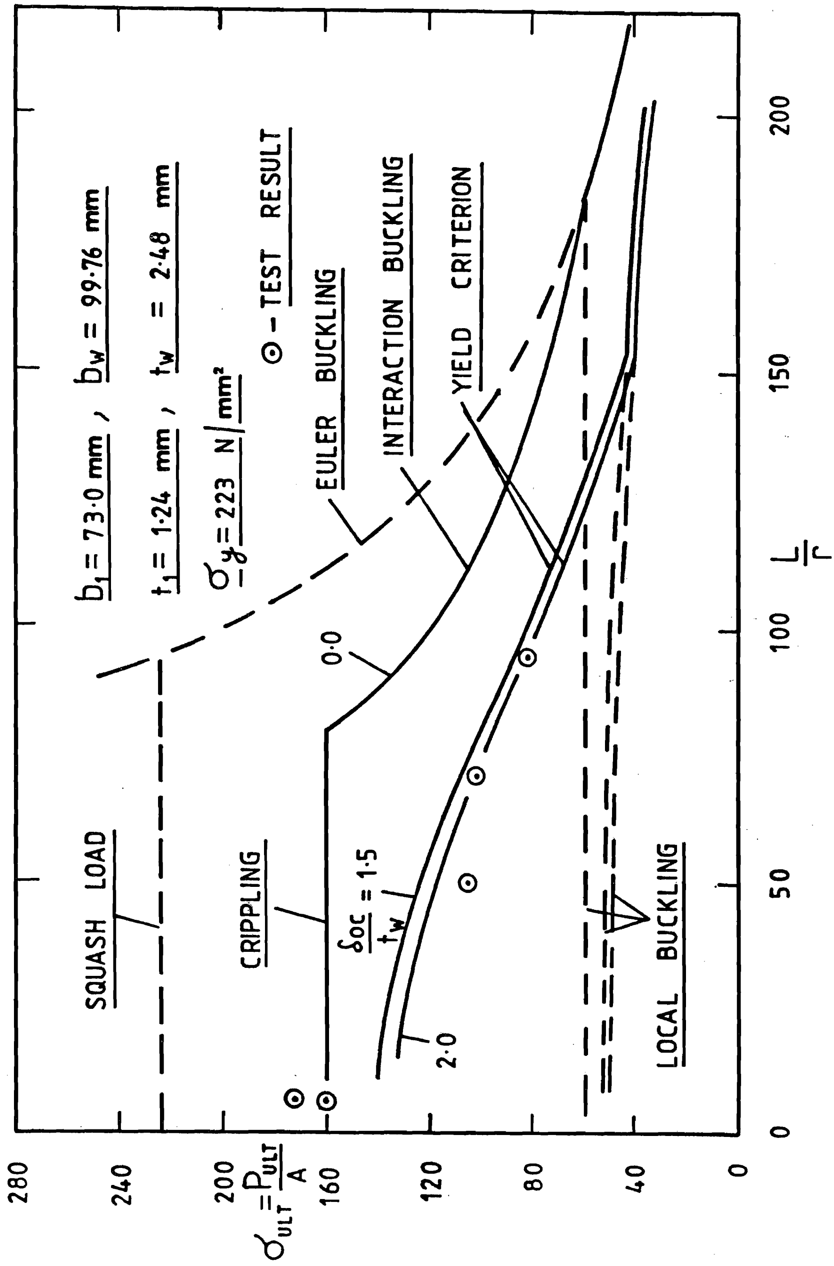

FIGURE 10. COMPARISON WITH SCI AND LCI TEST RESULTS OF REFERENCE 8.

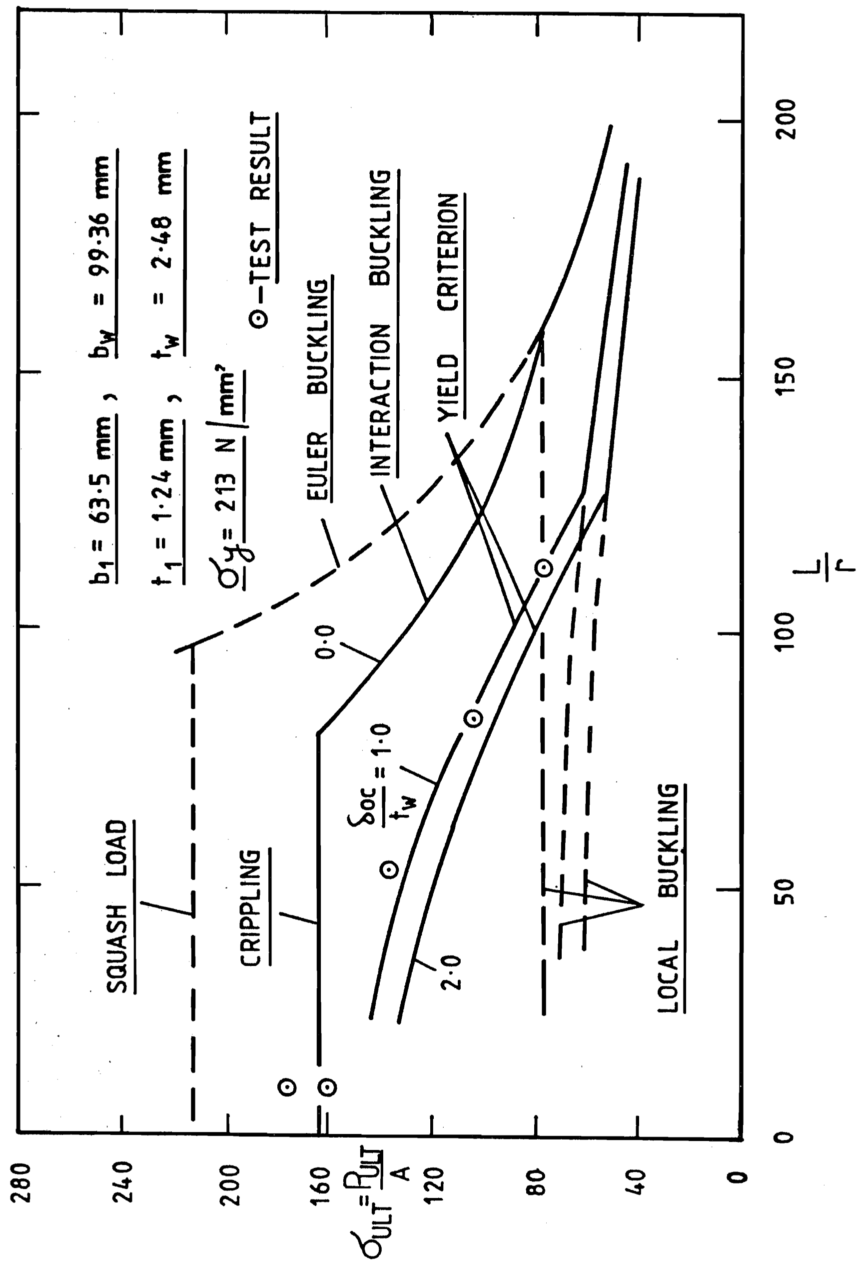

FIGURE 11. COMPARISON WITH SCII AND LCII TEST RESULTS OF REFERENCE 8.

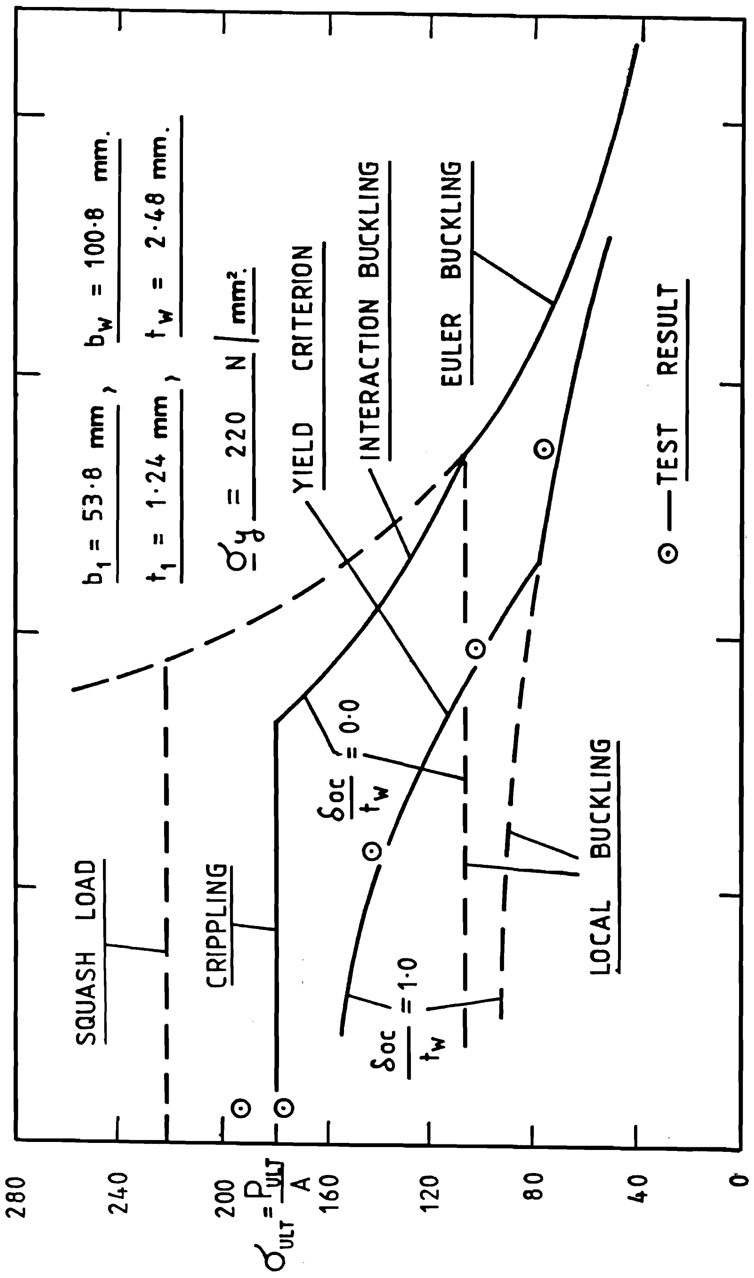

FIGURE 12. COMPARISON WITH SCIII AND LCIII TEST RESULTS OF REFERENCE 8.

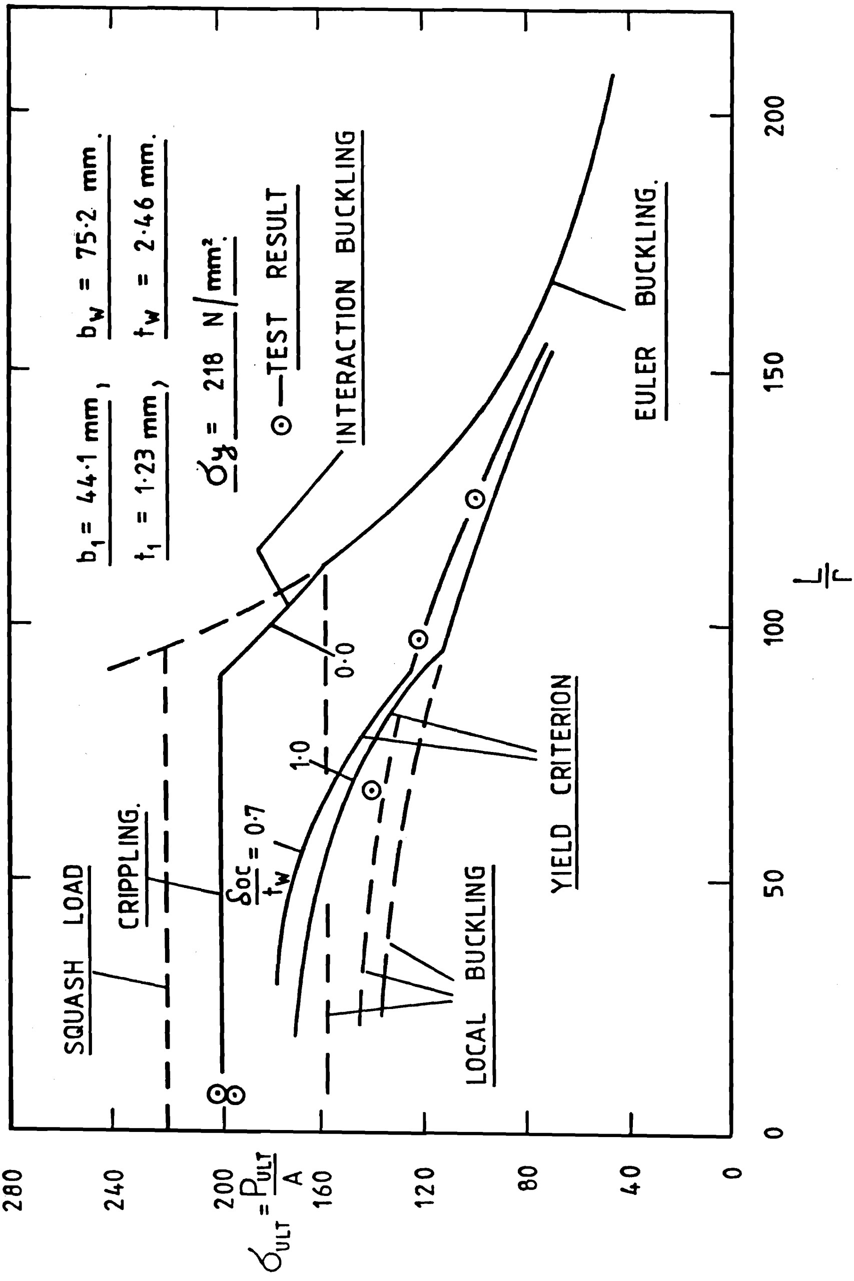

FIGURE 13. COMPARISON WITH SCIV AND LCIV TEST RESULTS OF REFERENCE 8.

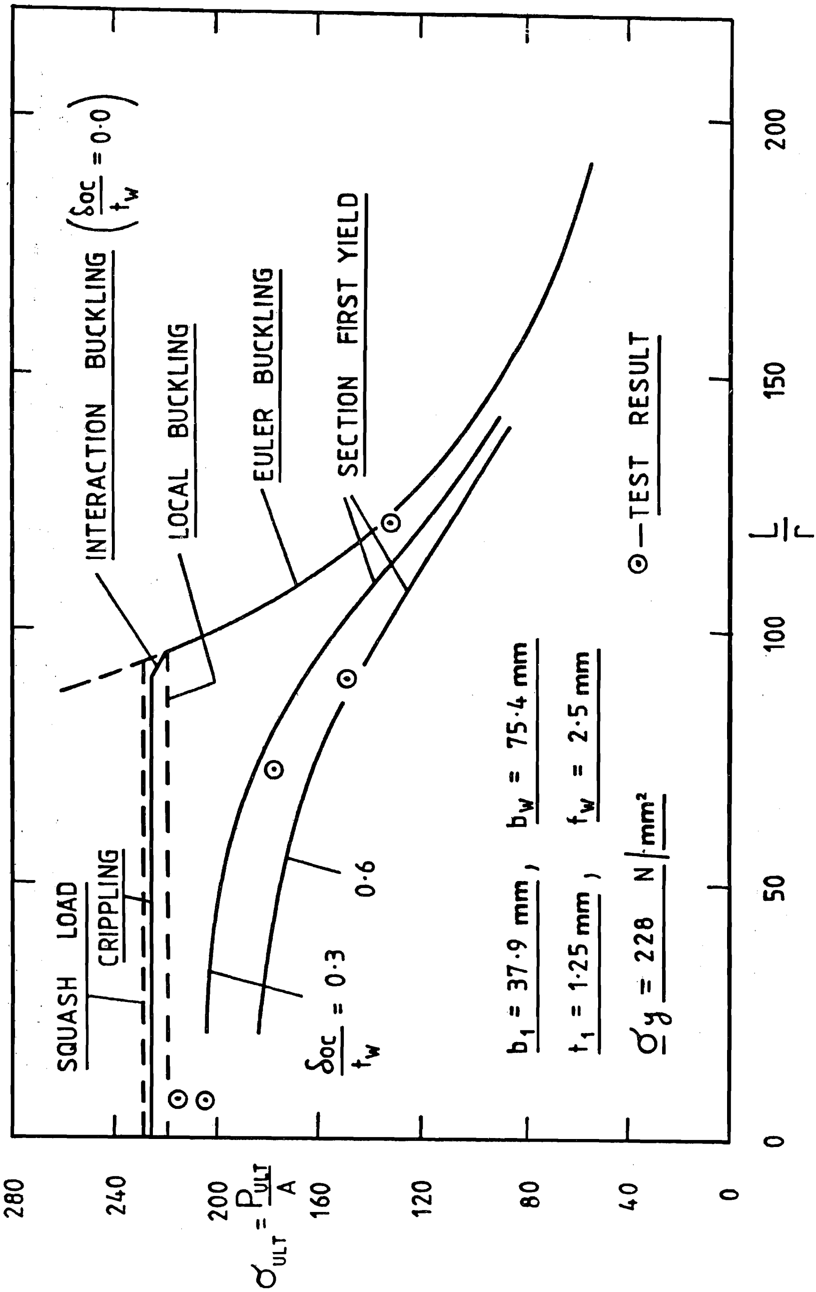

FIGURE 14. COMPARISON WITH SCV AND LCV TEST RESULTS OF REFERENCE 8.